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Technical Notes

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Unsteady Lifting Surface Theory in Sonic Flow: The Problem Revisited

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Introduction

DESPITE the steady transonic flow over wings being essentially nonlinear, the linearized potential results for unsteady motion appears to be meaningful whenever the maneuver presents a certain degree of unsteadiness and low-amplitude motion, as earlier observed by Bisplinghoff et al.[1] If one restricts the mathematical model to the potential flow equations, many methods of solution are available, at various degrees of approximations. See Ref. 2 for a complete review and discussion that introduces the subject of unsteady transonic potential flow over airfoils and wings.

The present investigation was stimulated by two facts. First, the statement by Garrick[3] that the existing means of analysis of nonstationary flow at sonic speed by linearized methods should not be disregarded because this constitutes a continuous bridging in analytical results from subsonic through transonic to supersonic speeds. Second, the possibility of expansion to the sonic range of a numerical method already applied in subsonic[4] and supersonic[5] wings in oscillatory flow.

Early attempts to solve the three-dimensional linearized transonic equation over lifting surfaces were made by Runyan and Woolston,[6] who extended the subsonic kernel function method to the Mach one limit, a box method based on a source distribution as described in Ref. 7, and the sonic box scheme due to Rodemich and Andrew,[8] composed of discrete rectangular panels of constant doublet density. The new proposed method of solution presented here follows Ref. 8. An important novelty is a new variable transformation, inspired by previous work,[4,5] leading to the classical diffusion equation of theoretical physics.[9] This allows evaluation of influence coefficients analytically, when integrands are singular, from the concept of an integral's finite part. Another remarkable difference is the numerical evaluation of influence coefficients when integrands are regular, in order to save computational time.

Problem Formulation in the Transformed Plane and Integral Equation

In a reference frame that translates uniformly with the undisturbed flow velocity U, close to the sound speed a, the complex velocity perturbation potential Φ due to the harmonic small-amplitude motion of a thin wing is described by a linear differential equation as given by Landahl[10] in the frequency domain:

$$\frac{\partial^2\Phi}{\partial Y^2}+\frac{\partial^2\Phi}{\partial Z^2}-\frac{2Ui\omega}{a^2}\frac{\partial\Phi}{\partial X}+\frac{\omega^2}{a^2}\Phi=0 \tag{1}$$

It is observed that U lies in the positive X direction and Φ is made nondimensional relative to U and a reference length L, the root semichord of the wing.

Defining now a new complex potential as

$$\phi=\Phi\exp[(i\omega X)/(2aM)] \tag{2}$$

where ω is the angular frequency of the motion and M is the undisturbed flow Mach number, and using the transformation

$$x=X/L,\qquad y=MY/L,\qquad z=MZ/L \tag{3}$$

the governing equation of the problem [Eq. (1)] can be rewritten as the classical diffusion equation,[9] just as was done for the subsonic[4] and supersonic[5] cases,

$$\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}-2ik\frac{\partial\phi}{\partial x}=0 \tag{4}$$

where $k=\omega L/U$ is the reduced frequency. Equation (4) is of a parabolic type and possesses source- and doubletlike elementary solutions.

The linearized boundary condition on the wing surface is written in the transformed plane as

$$w(x,y)=\frac{\partial\phi}{\partial z}=\frac{\exp(ikx/2)}{M}\left(\frac{\partial h}{\partial x}+ikh\right) \tag{5}$$

where $h(x,y)$ represents the wing surface nondimensional vertical displacement. The complex pressure coefficient is written as

$$Cp=\frac{-2}{UL}\exp\left(-\frac{ikx}{2}\right)\left(\frac{\partial\phi}{\partial x}+i\frac{k}{2}\phi\right) \tag{6}$$

and pressure continuity is ensured if Eq. (6) is applied to both sides of the wake, that is,

$$\delta Cp=0=\frac{\partial\delta\phi}{\partial x}+i\frac{k}{2}\delta\phi \tag{7}$$

where $\delta\phi$ and δCp are the complex velocity potential and pressure coefficient jump between the lower and upper surfaces of the wake, respectively. It is important to stress that the same condition applies whenever the trailing edges are subsonic.

The solution of the problem just described is obtained from the integral equation that relates the potential jump across the lifting surface (and wake) to the downwash. For a planar configurations this integral reads

$$w(x,y)=\frac{1}{4\pi}\iint\delta\phi\frac{ik}{(x-x_0)^2}\exp\left(-\frac{ik(y-y_0)^2}{2(x-x_0)}\right)\mathrm{d}x_0\,\mathrm{d}y_0 \tag{8}$$

The integrand represents a doublet at (x_0,y_0) inducing a normal velocity in the wing plane ($z=0$) at the receiving point (x,y). The integral sign must be taken in its usual way along y_0 and in the sense of the finite part integration along x_0. The general formulation relative to the integral equation (8) can be found in Ref. 9 and is a result of the application of Green's theorem to the diffusion equation (4). Its kernel function, the induced doublet velocity, comes from the unitary strength source velocity potential,

$$\phi(x,y,z)=\frac{1}{4\pi(x-x_0)}\exp\left(-ik\frac{(y-y_0)^2+(z-z_0)^2}{2(x-x_0)}\right) \tag{9}$$

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for a point source placed at (x_0, y_0, z_0), with $x > x_0$, which must be differentiated twice along the z direction. For $x \leq x_0$, $\phi = 0$.

Numerical Solution

Solutions of the problem are obtained by solving integral equation (8) for $\delta\phi$ using the boundary conditions (5) and (7) for the wing and wake surfaces, respectively. Both surfaces are discretized through the use of small rectangular elements of unknown constant density doublets, as for the case of the sonic Mach box formulation.[8] The boundary conditions are enforced at control points located at each panel geometrical center. The key features of the proposed method of solution are summarized in Fig. 1a. One can identify four kinds of integration domains in order to obtain the influence coefficients at P. Region I is completely upstream of the limiting Mach lines drawn from P, region II is only partially upstream of the limiting Mach lines, and region III is completely downstream of the limiting Mach lines, corresponding to zero influence. The fourth kind of integration domain corresponds to the self-induced influence coefficient, that is, panel IV at point P. In region I numerical integration is straightforward because the integrand is never singular.

By the consideration of a rectangular domain with span $2s$ (Fig. 1b) and for unitary $\delta\phi$, the influence coefficients are obtained, for regions II and IV, from double integration of Eq. (8). In variable y_0

$$F(x, y) = \frac{1}{4}\sqrt{\frac{ik}{2\pi}} \int \frac{\operatorname{erf}(\lambda_1) - \operatorname{erf}(\lambda_2)}{(x - x_0)^{\frac{3}{2}}} \, dx_0 \tag{10}$$

where

$$\lambda_{1,2} = (y \pm s)\sqrt{\frac{ik}{2(x - x_0)}} \tag{10a}$$

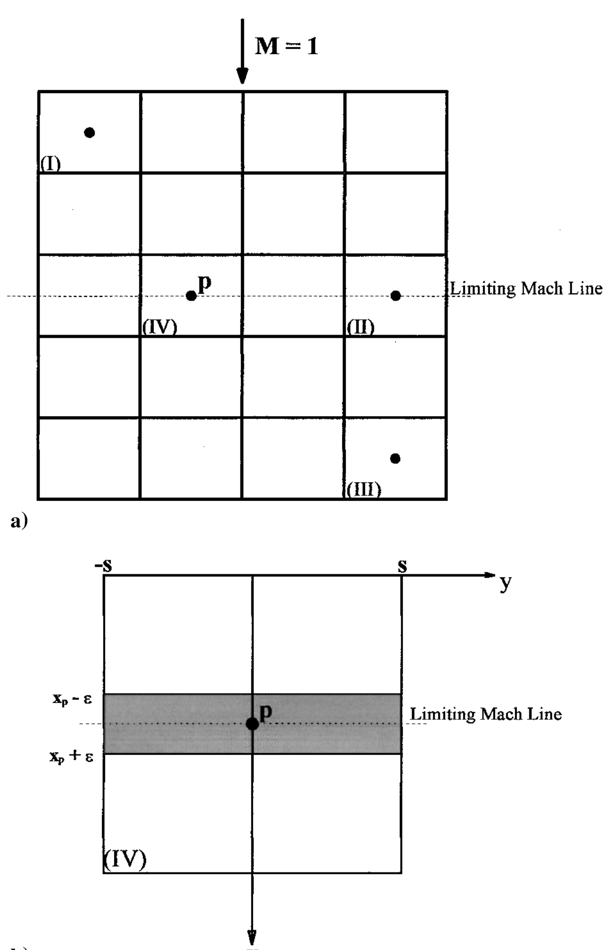


Fig. 1 Wing discretization a) integration domains for receiving point P and b) local panel reference frame for analytical integration.

which must be determined in the finite part sense as x tends to x_p. Integrating now along x_0 for x greater than x_p, one obtains

$$F(x, y) = \frac{1}{2\pi}\left(\frac{\exp(\lambda_3)}{(s + y)} + \frac{\exp(\lambda_4)}{(s - y)}\right) + \frac{1}{2}\sqrt{\frac{ik}{2\pi(x - x_0)}}[\operatorname{erf}(\lambda_1) - \operatorname{erf}(\lambda_2)] \tag{11}$$

where

$$\lambda_{3,4} = -\frac{ik(s \pm y)^2}{2(x - x_0)} \tag{11a}$$

To arrive at the final formula let us assume that k has a small negative imaginary part. For region IV, $y = 0$, and Eq. (10) becomes

$$F(x_p, 0) = \frac{1}{4}\sqrt{\frac{ik}{2\pi}} \int_0^{x_p} 2\operatorname{erf}\left(s\sqrt{\frac{ik}{2(x - x_0)}}\right) \Big/ (x - x_0)^{\frac{3}{2}} \, dx_0 \tag{12}$$

which must be evaluated as

$$F(x_p, 0) = \lim_{\varepsilon \to 0}\left\{[F(x, 0)]_0^{x_p - \varepsilon} + \frac{1}{2}\sqrt{\frac{ik}{2\pi}} \times \left[RP\int_{x_p - \varepsilon}^{x_p + \varepsilon} \frac{1}{(x - x_0)^{\frac{3}{2}}} \, dx_0\right]\right\} \tag{13}$$

since

$$\operatorname{erf}\left(s\sqrt{\frac{ik}{2(x_p - x_0)}}\right) \to 1 \tag{14}$$

as x_0 tends to x_p. The second term of Eq. (13) is the one to be considered in its finite part value as it was done in Ref. 11. For region II the given procedure is not necessary because signals of error functions in Eq. (11) are always symmetrical. The final expression for the influence coefficients that is valid for regions II and IV becomes, after letting the imaginary part of k go to zero,

$$F(x, y) = -\frac{1}{2\pi}\left(\frac{\exp(\lambda_8)}{(s - y)} + \frac{\exp(\lambda_7)}{(s + y)}\right) + \frac{1}{2}\sqrt{\frac{ik}{2\pi x_p}}[\operatorname{erf}(\lambda_6) - \operatorname{erf}(\lambda_5)] \tag{15}$$

where

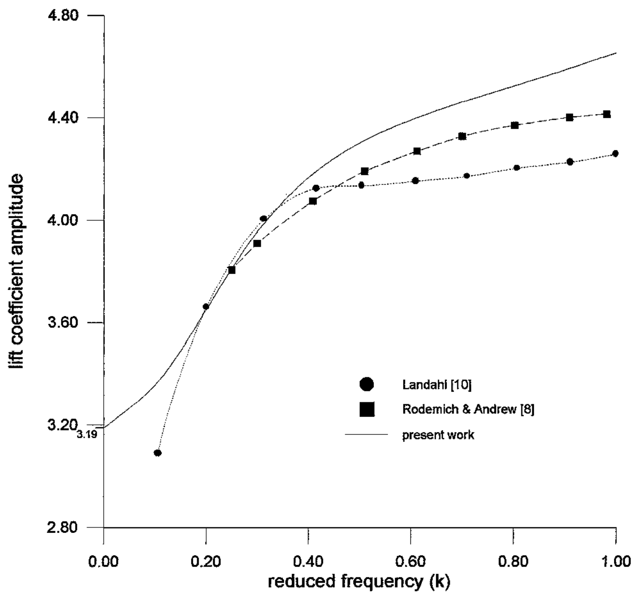
$$\lambda_{5,6} = (y \pm s)\sqrt{\frac{ik}{2x_p}}, \qquad \lambda_{7,8} = -\frac{ik(s \pm y)^2}{2x_p} \tag{15a}$$

The unsteady results using the present method are shown in Fig. 2.

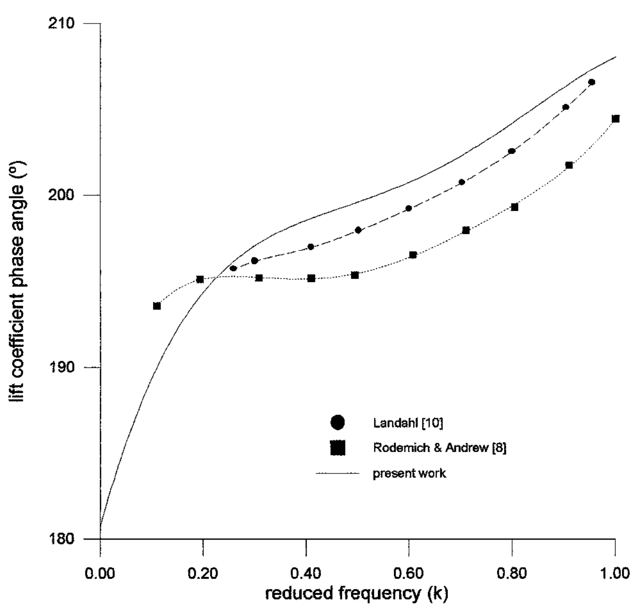
Results and Final Remarks

Numerical calculations for a rectangular wing of aspect ratio 2 are described and the results obtained are compared with those reported earlier by Landahl[10] and Rodemich and Andrew.[8] Landahl[10] uses series expansion for a small value of a Hankel function and the result is dependent on aspect ratio and reduced frequency. Rodemich and Andrew[8] present analytical formulas for the square box, but, nevertheless, some approximations are invoked.

Figure 2 shows unsteady results for a rectangular wing oscillating in pitch about its leading edge axis. Lift coefficient amplitude and phase angle compare fairly well with the ones given in Refs. 8 and 10. Note that the limit of the reduced frequency to zero corresponds to the steady-state solution and to the slender wing theory in this case. The values of lift coefficient and phase angle are both π if one takes into account the definition of phase angle employed in Refs. 8 and 10. For these calculations a grid of 32×32 square panels on the half-wing was used (25×25 was used in Ref. 8), and the computation time was less than 10 min on a Pentium 133 MHz, where most of the time is spent setting up the influence coefficient matrix. This computation time can be made significantly smaller if adequate approximation is employed for the complex error function calculations.



a) Lift coefficient amplitude



b) Phase angle of the lift coefficient

Fig. 2 Rectangular wing of aspect ratio 2 oscillating in pitch about the leading-edge axis.

Conclusion

This work presents a numerical method to solve the linearized sonic equation over lifting surfaces. For rectangular shapes one can admit that the solution is nominally exact because rectangular panels fit exactly the wing surface. For swept wings the model deserves further development, especially in terms of convergence behavior.

Acknowledgment

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Prediction of Hysteresis Associated with the Static Stall of an Airfoil

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Introduction

THE basic phenomenon of stall associated with airfoils is quite well understood and has now become standard textbook material. It is caused by massive flow separation resulting in sharp drop in lift and increase in the drag acting on the airfoil. In certain cases, hysteresis in the flow has been observed for angles of attack close to the stall angle. However, this phenomenon is not very well understood. Hoffmann[1] has reported the hysteresis loop in the data for aerodynamic coefficients for a NACA 0015 airfoil. He also studied the effect of freestream turbulence (FST) on the performance characteristics of the airfoil. The hysteresis in the data can be observed for low FST but disappears for high FST. The present work is an effort to study the behavior of the flow near stall by solving the governing flow equations numerically. Carefully conducted computations are utilized to track the hysteresis loop in the aerodynamic data close to the stall angle. To the best of the knowledge of these authors, this is the first effort of its nature. The incompressible, Reynolds-averaged Navier–Stokes (RANS) equations, in conjunction with the Baldwin–Lomax model[2] for turbulence closure are solved using stabilized finite element formulations. The finite element mesh consists of a structured mesh close to the body and an unstructured part, generated via Delaunay's triangulation, away from the body. This type of a grid has the ability of handling fairly complex geometries while still providing the desired resolution close to the body to effectively capture the boundary-layer flow, especially in the context of unsteady flows. Despite the simplicity of the Baldwin–Lomax model, its implementation with unstructured grids is not trivial. The interested reader is referred to the articles by Kallinderis[3] and Mavriplis[4] for details.

The finite element formulations and their implementations used in the present work are well proven and have been utilized to

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